A color-based selective and interactive filter using weighted TV

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Abstract. In this paper we propose to introduce a new color interactive and selective filtering tool based on the minimization of a weighted vectorial total variation term (TV_g) with the L^2 norm as data term. Our goal is to filter one region of an image while preserving the other. To this end, we introduce color moments in the TV_g term. Up to now, color moments have been mainly introduced for indexation purposes. In our case, the user selects some points in the area to preserve and some other points in the area to be filtered. Reference color moments are then computed on patches around the selected points and are included in the TV_g term. Two main functions g are tested within the HSL color space leading to interesting results on both synthetic and real images. Convex optimization tools are used to solve the minimization issue and we take benefit of an augmented Lagrangian formulation and Uzawa block relaxation schemes. The proposed filtering tool may be interesting as a pre-processing step for segmentation, movie post-production or object-oriented compression.

Keywords: color filtering, moments color, total variation, convex optimization, augmented Lagrangian

1 Introduction

In many image processing issues, a filtering step is required in order to remove noise or spurious details from the initial image. The image may then be decomposed in regions (e.g. homogeneous regions) using a segmentation algorithm, or some regions of interest may be selected using well-defined algorithms (active contours for example). The interactive color filtering tool proposed in this paper is quite different from a simple filtering step and may even be considered as a pre-segmentation algorithm. Indeed, the main idea is to filter and simplify some parts of the image while preserving some others using color properties of the regions. In order to obtain such a filtering tool, we focus on the minimization of functionals that take benefit of a color weighted total variation (TV_g) regularization term coupled with the minimization of a L^2 data term. Indeed, the total variation (TV) regularization term owns some interesting geometrical properties [19, 1, 7], that can be nicely used for denoising and segmentation. The TV regularization term was first proposed for denoising in [20] coupled with the minimization of the $\mathbf{L^2}$ norm as a data term (the well-known ROF model). More recently, Bresson et al [1] proposed to introduce a spatially adaptive TV term using a function g inside the integral of the TV term. The function g allows to take into account the image gradient in the regularization term in order to preserve object boundaries during the denoising process. Some other authors also take benefit of such a weighted TV [5, 13, 1, 14, 16, 17, 6].

In this paper, we propose to settle an interactive color image filtering algorithm by introducing color moments in the weighted TV regularization term. The user is expected to draw a curve on the region to be removed and another curve on the region to be preserved. The color moments of the chosen points are then computed on small patches and the corresponding characteristics are included in a well-adapted g function in the TV_g term. As far as color moments are concerned, they are considered as a powerful color descriptor. They have been introduced in [18] for image indexation and retrieval in order to complete or

replace SIFT descriptors. They appear to be more interesting than the classical color histograms used in [9, 11, 15]. Indeed, color histograms are not able to represent the spatial layout of the color repartition. In an original manner, we propose to take advantage of such a representation for our selective color filtering tools. Concerning the optimization of the whole functional ($TV_g + L^2$), we rely on the mathematical framework previously introduced in [16] for grey level images and in [17] for color images. The optimization of the vectorial $TV_g + L^2$ is based on an augmented Lagrangian approach and Uzawa block relaxation schemes. Our numerical scheme presents the advantage to be robust regarding with the choice of the penalty parameter and is efficient in terms of computational cost. The proposed filtering tool may be very interesting as a pre-processing step for segmentation, movie post-production or object-oriented compression.

In the rest of the paper, we first detail the main principles of our selective filtering tool in section 2. The optimization process is detailed in section 3 and the color-based weighted TV is introduced in section 4. Finally some experimental results on both synthetic and real images are given in section 5.

2 Selective filtering based on the minimization of a weighted TV

2.1 Geometrical properties of TV and weighted TV

The classical ROF (Rudin, Osher and Fatemi) model [20] aims to recover the original image $u(x)$ given a noisy image $f(x)$ by minimizing the total variation under L^2 data fidelity:

$$E(u) = \int_\Omega |\nabla u(x)| \, dx + \lambda \int_\Omega (u(x) - f(x))^2 \, dx, \tag{1}$$

where $\Omega \subset \mathbb{R}^2$ is the image domain and λ a positive scale parameter.
The first integral is the classical regularization term called TV (Total Variation) term. This term has proven its efficiency for image restoration and also presents some interesting geometrical properties. In order to better explain these properties, let us denote the upper level sets of the image by $U^\alpha(u) = \{x, u(x) > \alpha\}$ like in [4]. From a geometrical point of view, the co-area formula [8] states that, for any function which belongs to the space of bounded variations BV(Ω), there is a relation between the TV regularization term and the perimeter $Per(U^\alpha)$ of the set U^α. We can write $Per(U^\alpha(u)) = \int_\Omega |\nabla \chi_{U^\alpha(u)}| \, dx$ for all α where $\chi_{U^\alpha}(u)$ stands for the characteristic function of the set $U^\alpha(u)$. Such geometrical features may contribute to explain the properties of this regularization term. Indeed, when decreasing the weight λ of the data term, components will be removed in an order determined by their size and their geometry. For example, small components will be removed first and sharp angles will be smoothed. In [7], the authors establish a connection between this model and morphological operators such as opening and morphological granulometry.

The introduction of a g function in TV may produce different filtering results. Indeed, the TV_g term, when applied to a characteristic set is equivalent to a weighted perimeter $\int_C g(s) ds$ where C designates the boundary of the set and s its arc length. In [1], the function g is then chosen as $g(x) = 1/(1 + \beta G_\sigma * |\nabla f|)$ in order to introduce the image gradient directly in the regularization term (β is a positive constant and $G_\sigma *$ represents the convolution with a Gaussian of variance σ). This term allows to preserve object boundaries and sharp angles during the regularization process and can also be used for shape segmentation [1]. Indeed, this regularization term corresponds to the classical criterion proposed in geodesic active contours [2]. Note that such a spatially varying TV term has also been investigated by different authors with various g functions (e.g. for salt and pepper denoising) [5, 13, 1, 14, 16, 17, 6].

2.2 Color selective filter using weighted TV

In this paper, we propose to test the availability of the weighted TV regularization term in order to perform a kind of selective filtering of the image components. This idea is closely related to the design of geometrical

filters in the framework of mathematical morphology where some shapes are removed on the basis of their geometric properties. Some examples of geometrical filtering using the TV_g term are given in [16, 17]. Rather than using geometric properties, we here propose to take benefit of color moments and of a vectorial TV term in order to include some color features in the function g. In order to introduce a selective filtering scheme, we add an interactive step where two regions are manually selected (using a curve drawn on each region). The color moments of each point within each region are computed leading to a reference vector of moment for each region. The function g is then designed in order to filter one region while preserving the other one and inversely. The main principle of this scheme is summarized in Fig.1.
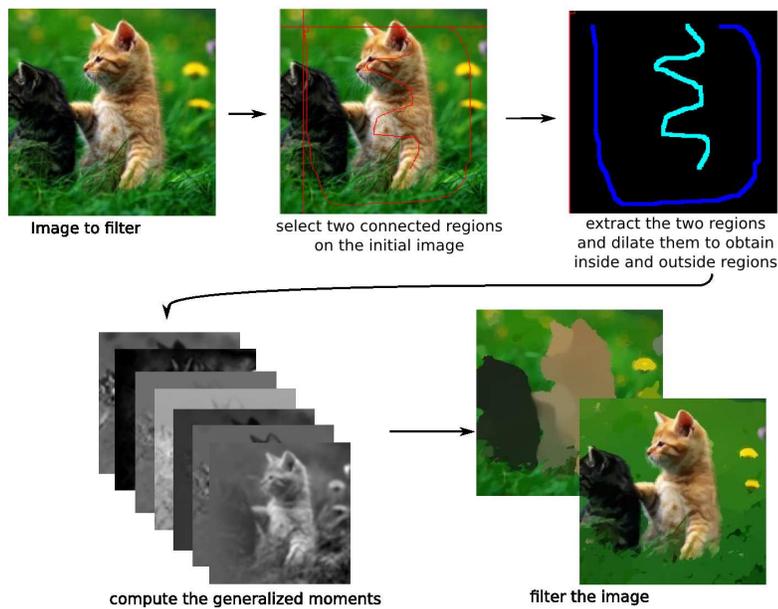

Fig. 1. Main principles of the proposed color-based interactive and selective filtering tool.

3 Fast dual minimization of $TV_g + L^2$

We define Ω as the image domain $\subset \mathbb{R}^d$, $d = 2, 3$ and $\mathbf{u}(x) = (u_1(x), u_2(x), u_3(x)) \in \mathbb{R}^3$ as a vector-valued function defined on Ω that corresponds to the color intensity with the three values of each color channel. In the rest of the document, vector valued functions are denoted by bold-face letters (e.g. $\mathbf{u} = (u_1, u_2, u_3)$). Let us also note the Euclidean scalar product by $\mathbf{u} \cdot \mathbf{v} = \sum_{i=1}^{3} u_i v_i$, for $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^d$. Moreover, for $\mathbf{u} \in \mathbf{R}^d$, we use the notation $|\mathbf{u}|_2 = (\mathbf{u} \cdot \mathbf{u})^{1/2}$ for the Euclidean norm.

Let g be a continuous, positive valued and bounded function defined on Ω, we consider the following weighted total variation regularization term, denoted by TV_g :

$$TV_g(\mathbf{u}) = \int_{\Omega} g(x) \left[|\nabla u_1|_2^2 + |\nabla u_2|_2^2 + |\nabla u_3|_2^2\right]^{1/2} dx.$$

The considered function g is described later in section 4.

Let $\mathbf{f} = (f_1, f_2, f_3)$ be the input color image to be filtered, we propose to address the following vectorial $TV_g + L^2$ minimization problem:

$$\min_{\mathbf{u} \in \mathbf{X}} E(\mathbf{u}) = J(\mathbf{u}) + \lambda \int_\Omega |\mathbf{u}(x) - \mathbf{f}(x)|_2^2 \, dx. \tag{2}$$

where $\mathbf{u}$ is the unknown image to restore and $\mathbf{X}$ is a suitable functions space.

3.1 Augmented Lagrangian methods for the $TV_g + L^2$ model

In this section we propose to remind the convex optimization tools used in order to solve (2). This framework is issued from our previous works notably [16] where the resolution is performed for grey level images, and [17] where the vectorial $TV_g + L^\tau$ model has been solved for vectorial images with $\tau = 1$ or 2. We remind here only few elements for completeness. The convex minimization problem (2) is transformed into a suitable saddle-point problem by introducing an auxiliary unknown. An augmented Lagrangian approach is then introduced and solved using Uzawa relaxation schemes. Some details are given thereafter for the reader convenience but only briefly for space reasons.

Let us introduce the auxiliary unknown $\mathbf{p} = \mathbf{f} - \mathbf{u}$ and rewrite the functional E as

$$E(\mathbf{u}, \mathbf{p}) = TV_g(\mathbf{u}) + \lambda \int_\Omega |\mathbf{p}(x)|_2^2 \, dx. \tag{3}$$

The minimization problem (3) becomes $\min_{(\mathbf{u},\mathbf{p}) \in K} E(\mathbf{u}, \mathbf{p})$,, where the constraint set K is defined by $K = \{(\mathbf{u}, \mathbf{p}) \in X \times X \mid \mathbf{u} + \mathbf{p} - \mathbf{f} = 0 \text{ in } \Omega\}$. With this constrained minimization problem, we associate the Lagrangian functional $\mathscr{L}$ defined on $\mathbf{X}^3$ by

$$\mathscr{L}(\mathbf{u}, \mathbf{p}; \mathbf{s}) = E(\mathbf{u}, \mathbf{p}) + (\mathbf{s}, \mathbf{u} + \mathbf{p} - \mathbf{f})_{\mathbf{X}}. \tag{4}$$

In (4), $\mathbf{s}$ is the Lagrange multiplier associated with the constraint in K. We now introduce the augmented Lagrangian defined, for $r > 0$, by

$$\mathscr{L}_r(\mathbf{u}, \mathbf{p}; \mathbf{s}) = \mathscr{L}(\mathbf{u}, \mathbf{p}; \mathbf{s}) + \frac{r}{2} \| \mathbf{u} + \mathbf{p} - \mathbf{f} \|_{L^2}^2 \tag{5}$$

where r is the penalty parameter.

3.2 Uzawa block relaxation methods

We apply an Uzawa block relaxation method by using an alternative minimization procedure (see e.g. [10, 12]). Such algorithms are fast, easy to implement, and also robust to the choice of the penalty parameter which allows to choose a value of r that minimizes the number of iterations and so the computational cost [17]. Given $\mathbf{p}^{-1}$ and $\mathbf{s}^0$, we compute successively $\mathbf{u}^k$, $\mathbf{p}^k$ and $\mathbf{s}^{k+1}$ as follows:

4 Proposition of a weighted TV term based on Color moments

Let us now introduce the weighted TV regularization designed for our color selective filtering tool. Given a color image $\mathbf{u} = (u_1, u_2, u_3)$, we propose to take benefit of generalized color moments M_{pq}^{abc} introduced in [18]. The color moment of order $p + q$ and degree $a + b + c$ is defined as follows:

$$M_{pq}^{abc} = \int \int_D x_1^p x_2^q [u_1(x_1, x_2)]^a [u_2(x_1, x_2)]^b [u_3(x_1, x_2)]^c \, dx_1 dx_2 , \tag{7}$$

Algorithm 1 Uzawa block relaxation algorithm

Initialization. $\mathbf{p}^{-1}$, $\mathbf{s}^0$ and $r > 0$ given.

Iteration $k \geq 0$. Compute successively $\mathbf{u}^k$, $\mathbf{p}^k$ and $\mathbf{s}^k$ as follows.

    Step 1. Set $\tilde{\mathbf{p}} = \mathbf{s}^k + r(\mathbf{p}^{k-1} - \mathbf{f})$, compute $\mathbf{v}^k$ and then $\mathbf{u}^k = \mathbf{f} - \mathbf{p}^{k-1} + \dfrac{1}{r}(\nabla \cdot \mathbf{v}^k - \mathbf{s}^k)$.

    We can compute $\mathbf{v}^k$ using the following semi-implicit scheme due to Chambolle [3]:

$$v_i^{\ell+1} = \frac{v_i^\ell + \tau \nabla(\nabla \cdot v_i^\ell - \tilde{p}_i)}{1 + (\tau/g)\left[\sum_{i=1}^3 |\nabla(\nabla \cdot v_i^\ell - \tilde{p}_i)|_2^2\right]^{1/2}}, \quad i = 1, 2, 3, \quad \text{where} \quad \tau > 0 \tag{6}$$

    Step 2. Update the Lagrange multiplier: $\mathbf{s}^{k+1/2} = \mathbf{s}^k + \dfrac{r}{2}(\mathbf{u}^k + \mathbf{p}^{k-1} - \mathbf{f})$.

    Step 3. Compute $\mathbf{p}^k = -(\mathbf{s}^k + r(\mathbf{u}^k - \mathbf{f}))/(r + 2\lambda)$

    Step 4. Update the Lagrange multiplier: $\mathbf{s}^{k+1} = \mathbf{s}^{k+1/2} + \dfrac{r}{2}(\mathbf{u}^k + \mathbf{p}^k - \mathbf{f})$.

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where $x = (x_1, x_2)$ is a pixel in a 2D image and a, b, c, p and q some positive integers and D the moment computation domain (it can be the whole image or a patch).

In order to use a semi-local information on color (which can be interesting when considering texture regions), we propose to compute color moments on small patches around each pixel of the considered region. We denote this small neighborhood by D_n where n represents the half size of the neighborhood. Note that the moments of degree 0 correspond simply to the classical geometric moments of D_n and so they do not provide useful information for filtering since they only encode the fixed geometry of D_n. In our application we consider 27 moments which are M_{00}^{abc}, M_{01}^{abc} and M_{10}^{abc} with $(a, b, c) \in \{(1,0,0); (0,1,0); (0,0,1); (1,1,0); (1,0,1); (0,1,1); (2,0,0); (0,2,0); (0,0,2)\}$. The corresponding 27-vectors of moments are noted $\mathbf{M}(x)$ where $x \in \Omega$. In our application we consider the HSL color space which conducts in practice to better results than the RGB color space. Note that the space color can be changed easily using a simple conversion at the beginning of the process.

The semi-supervised filtering scheme has been previously described in Figure 1. In the first step, we draw two curves on the image, one for the "inside" region and the other for the "outside" region. The "outside" region Ω_{out} will be filtered and the "inside" one Ω_{in} remains without filtering. A dilatation of what we can call "the markers" is performed and the color moments for each selected region are then computed. For the dilatation step, we use a square of half size of 2 pixels as a structuring element. We present thereafter two main solutions for the design of a weighted TV term taking into account these color moments.

4.1 Definition of weighted TV based on the average moment vector

The most straightforward method is to compute an average moment vector of the markers Ω_{in} and Ω_{out} (averaging each component of the vector). The average value for the component α is $\mu_i^\alpha = \frac{1}{|\Omega_i|} \int_{\Omega_i} M_\alpha(x_1, x_2) dx_1 dx_2$ with $\alpha \in [0, 27]$ and $i = in$ or out. We then propose to select the order α^* that corresponds to the maximum distance between the two average values as follows :

$$\alpha^* = \max_{\alpha \in [0,27]} \left(|(\mu_{in}^\alpha - \mu_{out}^\alpha)| \right), \tag{8}$$

We then propose to introduce the following thresholding function :

$$T(x) = \begin{cases} c_1 \ if \ M_{\alpha^*}(x) < \mu_{\alpha^*} \\ c_2 \ if \ M_{\alpha^*}(x) \geq \mu_{\alpha^*} \end{cases} \tag{9}$$

where μ_{α^*} corresponds to the average of the moments M_{α^*} of order α^* of the whole image Ω and is then chosen as a threshold. We choose $c_1 = 0.01$ and $c_2 = 1$ or inversely in order to uppermost smooth the pixels corresponding to the reference region r_{out} or r_{in}. A regularized continuous version of the function $T(x)$ is needed which is obtained by a Gaussian filtering: $g(x) = G_\sigma * T(x)$ ($\sigma = 0.05$).

4.2 Definition of a weighted TV using $1 - nn$ selection

The previous method is not able to deal with large variations of color inside each reference region. In order to cope with this problem, we propose to consider the whole 27 vector as a descriptor. We then compute the L_2 distance between the 27 moments of pixel $\mathbf{x}$ and the 27 moments of each pixel of the inside and outside region and we select the nearest pixel in each reference region. So, for each pixel x of the image, we choose the pixel $\mathbf{x}^*$ in the reference region r_i ($i = in$ or out) such that : $x_i^* = \arg\min_{\mathbf{y} \in r_i} \left[\sum_\alpha (M^\alpha(x) - M^\alpha(y))^2 \right]$. For each pixel x, the pixel x_i^* designates the pixel in the region r_i with the nearest moment values to x in the sense of the L^2 norm.

A thresholding function is then defined using $T(x) = c_1$ if $x_{in}^* \in r_{in}$ and c_2 if $x_{in}^* \in r_{out}$. We choose $c_1 = 0.01$ and $c_2 = 1$ in order to uppermost smooth the pixels corresponding to the reference region r_{out} while preserving r_{in}. The role of r_{in} and r_{out} can be inverted to preserve r_{out}. A regularized continuous version of the function $T(x)$ is needed which is obtained by a Gaussian filtering: $g(x) = G_\sigma * T(x)$ ($\sigma = 0.05$).

5 Experimental results

To test our approach, we conduct tests on synthetic and natural images using the two different g functions (μ and $1 - nn$). The patch size is chosen as $n = 3$ which represents a good trade-off between precision and quantity of information (a higher patch size can be needed in order to filter texture with larger patterns or to filter images with a higher resolution). The results are depicted in Fig.2 and Fig.3. In each figure, we both display the two results obtained when choosing r_{in} or r_{out} to be preserved.

The comparison between μ and $1 - nn$ is straightforward. The $1 - nn$ method outperforms significantly the other one showing the interest of designing a well-adapted function g. This can be easily explained since μ only compares the average of the whole selection to the current moment of the pixel. The $1 - nn$ method selects interactively the nearest vector of moments inside each region and then the nearest region (inside or outside) to apply the filtering. So it delivers a more local descriptor of the selected regions.

In order to get some quantitative results on the accuracy of the object contour in the filtered image, we propose to filter an image with an available reference segmentation. We take the leaf image and the reference segmentation mask respectively displayed in Fig.4.(a) and (b). We then perform our filtering algorithm using different parameters and we perform a simple threshold on the image in order to get the object without the background. The mask obtained after filtering O is then compared to the reference mask O_{ref} using the Dice coefficient ($DC(O, O_{ref}) = \frac{2|O \cap O_{ref}|}{|O| + |0_{ref}|}$) and the number of false positive and false negative pixels. The results are displayed in Table 1 for two different resolutions of the same image and for different values of the half patch size. Of course when dealing with a higher resolution, the mask is closer to the reference segmentation mask (the value of DC is near 1). The number of FP increases with the patch size while the number of FN decreases. This can be explained by the fact that the contour is better defined with a small size patch. On the contrary, choosing a small patch size leads to less robust description of the selected region and then increases the number of FN pixels. The best trade-off seems to occur for a half patch size of 3 for the two resolutions regarding with these quantitative parameters. Since the results give a strong smoothing effect on the outside

region while preserving all the details of the inside one, this method may give a way to perform an object-oriented compression. The algorithm can also be used as a pre-processing step for different applications such as segmentation or object selection or even artistic post-production of images or movies.
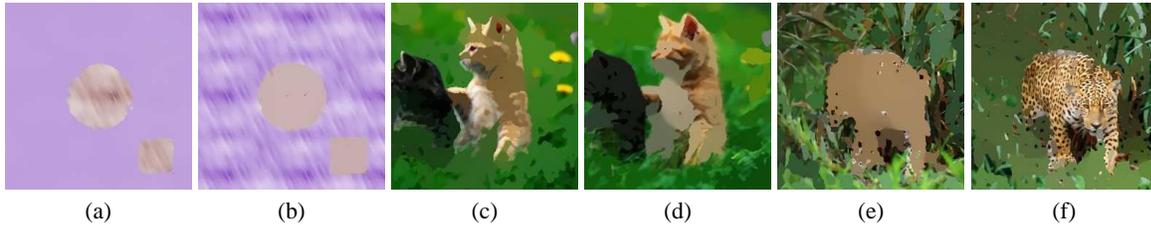

(a) (b) (c) (d) (e) (f)

Fig. 2. Experimental results on a synthetic image and on two natural images (selection and filtering of the background in (a) and of the geometric components in (b), selection and filtering of the two cats in (c) and of the background in (d), selection and filtering of the leopard in (e) and of the background in (f)) using μ approach and HSL space.
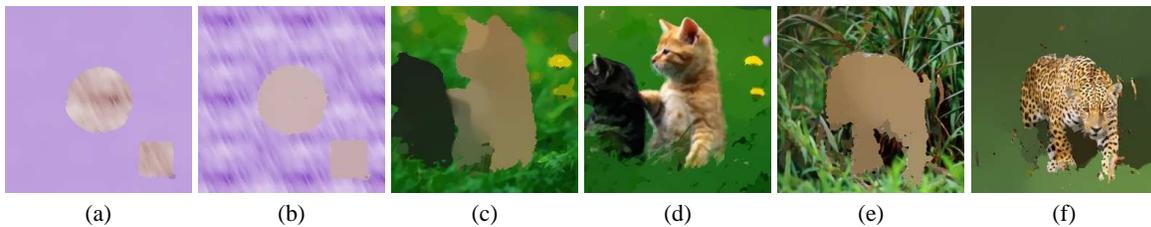

(a) (b) (c) (d) (e) (f)

Fig. 3. Experimental results on a synthetic image and two natural images (selection and filtering of the background in (a) and of the geometric components in (b), selection and filtering of the two cats in (c) and of the background in (d), selection and filtering of the leopard in (e) and of the background in (f)) using $1 - nn$ approach and HSL space.

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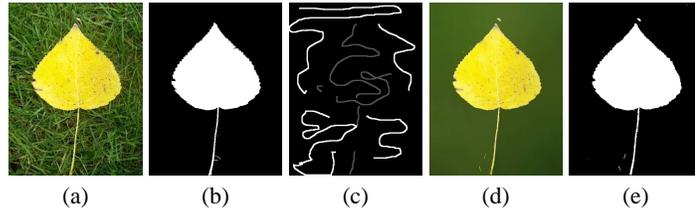(a)   (b)   (c)   (d)   (e)

Fig. 4. The image (a) represents the initial image and (b) the reference segmentation mask. The image (c) corresponds to the selected components of the two regions after the dilatation process, the image (d) to the filtering result using $1 - nn$ approach and HSL space ($\lambda = 0.1$ and $n = 3$) and the image (e) to the mask obtained from (d) after a simple thresholding.

Resolution: $643 * 496$					Resolution: $321 * 248$			
n	1	3	5	7	n	1	3	5	7
DC	0.989	0.990	0.989	0.988	DC	0.984	0.986	0.986	0.982
FP	770	898	1043	1348	FP	176	230	332	508
FN	811	487	376	267	FN	377	249	157	99

Table 1. Computation of the Dice Coefficient (DC), the number of false positive pixels (FP) and the number of false negative pixels (FN) according to the half patch size for two resolutions of the leaf image.

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